Common Core Standards for Mathematical Practice	Carnegie Learning Interpretation of Standard	Citations
INTRODUCTION The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).	Carnegie Learning believes the teacher's role is critical in the development of students' proficiency across all standards for mathematical practice. As a facilitator, the teacher is responsible to create daily opportunities and establish norms that allow students to develop mathematical understanding from prior knowledge, build connections, and foster each student's accountability to think, reason, and explain. Expertise is a long-term goal and students must be shown how to apply these practices to new content throughout their high school career. Each lesson provides opportunities for students to think, reason, and communicate their mathematical understanding. However, it is the responsibility of the teacher to recognize these opportunities and incorporate these practices into their daily rituals. We believe that effective communication and collaboration are essential skills of the successful learner. It is through dialogue and discussion of different strategies that students become knowledgeable, independent learners.	Carnegie Learning's pedagogical approach focuses on how students think, learn, and apply new knowledge in mathematics. This approach is consistent with the Common Core State Standards for Mathematical Practice. Each practice is developed throughout each course through a combination of instructional design and teacher implementation. The citations below are single instances of how we develop each mathematical practice. But, they are by no means the only instances.

1. Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	Carnegie Learning believes that our implementation model (discuss, think, pair, share) provides a classroom environment for students to make sense of problems, develop strategies, persevere in implementing the strategy, and analyze the results. At the beginning of each lesson, teachers are encouraged to engage in a class discussion to ensure that all students understand the problem presented and help students look for entry points to its solution. As students work collaboratively through problems, they plan and execute a solution strategy. It is the responsibility of each member of the group to monitor and evaluate the progress of the group and make suggestions for changing course, if necessary. As a facilitator, the teacher moves from group to group monitoring student work, assessing progress, and redirecting with guided questions. To bring closure and provide a summary for each problem, it is the intent that teachers ask thought-provoking questions that require students to explain their thinking and process. Multiple groups present their solutions and the class discussion centers around the use of alternate solution paths, connections to prior concepts, and generalization.	The Mathematical Representation page describes in detail Carnegie Learning's overarching philosophy as students solve problems and work with multiple representations of mathematical ideas. This sequence of icons appears consistently in every lesson throughout each course. Through the process of: • Discuss to Understand • Think for Yourself • Work with Your Partner • Work with Your Group • Share with the Class students will develop the capacity to make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	The scenarios within each lesson helps students recognize and understand that the quantitative relationships seen in the real world are no different than the quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.	**IN A2 1.1** In Problem 1, students make use of their intuitive understanding of money to create a table and develop an informal understanding of arithmetic sequences. In questions 5-8, students compare and contrast arithmetic and non-arithmetic sequences to develop an informal understanding of common difference. Students are then presented with the formal definition of an arithmetic sequence. After being introduced to explicit and recursive formulas for arithmetic sequences, students apply the formulas in context in Question 17.

3. Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	Carnegie Learning believes in a student-centered classroom, as opposed to one that is teacher- directed. In this type of classroom environment students are active participants in their learning; they are doing the work, presenting solutions, and critiquing each other. The teacher's role is to facilitate the discussion and highlight important connections, strategies, and conclusions. Often students only focus or mentally engage with a problem when they're required to produce a "product" or "answer." Carnegie Learning believes that the process is more important than the end result. The over-arching questioning strategy throughout the text is to promote analysis and higher order thinking skills beyond simple "yes" or "no" responses. By explaining problem-solving steps or the rationale for a solution, students will internalize the processes and reasoning behind the mathematics. Each lesson ends with the statement "Be prepared to share your answers with the class." Students are expected to be able to communicate their reasoning and critique the explanation of others.	**IN A2 1.1** The question phrasing in Problem 1 requires deeper thinking and reasoning: • Describe how to calculate your cousin's earnings based on the number of students in the tutor group. • Describe how to determine the next number in the earnings column knowing only the previous number. • Look at the following sequences of numbers. For each sequence, list the next three terms and describe how each new term was generated. • Describe the general pattern for determining the next term, given the previous term, in all four cases in Question 5. • How are the patterns you described for the sequences in Question 5 different from the patterns you described for the sequences in Question 7? • Is it reasonable to interpret the common difference of an arithmetic sequence as the slope? Explain. • Is this sequence arithmetic? How do you know?

4. Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	Throughout lessons, students create and use multiple representations (tables, graphs, symbolic statements, and words) to organize, record, and communicate mathematical ideas. Inherent in a collaborative classroom, students develop solutions in their group that leverage various models. As part of the classroom discussion, students may have to revise or update their group's model and solution with a more appropriate, efficient, or correct model or solution. The consumable textbook provides a record of the final solution, including appropriate models and reasoning.	**IN A2 12.5** Students model scenarios using tables, graphs, and circular functions. Within their groups, students agree upon appropriate models and improve upon or revise models as necessary.

5. Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	Carnegie Learning believes in the appropriate use of tools throughout the curriculum. Specifically, graphing calculator use is encouraged when appropriate in classroom instruction, both to determine a solution as well as to check results. Teachers should consider issues such as equity, technology policies for assessments, and the relative importance of symbolic manipulation when making instructional decisions about graphing calculator usage in the classroom. Graphing calculator instructions are provided as appropriate within lessons. Carnegie Learning Blended Math Curricula offer a combination of collaborative, student-centered textbook lessons and adaptive Cognitive Tutor software lessons.	**IN A2 14.1** Students use graphing calculators to generate random numbers. **IN A2 15.2** Students use graphing calculators to calculate a binomial probability. **IN A2 15.5** Students use graphing calculators to determine the area under the standard normal curve below a z-score.

6. Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	Carnegie Learning's intends that students communicate with precision when writing in their consumable books and sharing their solutions and methods with their peers. It is our expectation that students label units of measure, explain their reasoning throughout using definitions and appropriate mathematical language. However, it is the responsibility of the teacher to ensure that this intent is enforced. The answers provided in the Teacher's Implementation Guide are exemplars of student responses and model precision appropriately. A Vocabulary section is included in the Skills Practice for each lesson that provides additional opportunity to use language and terminology appropriately.	**IN A2 all lessons** Carnegie Learning has the expectation that student communicate precisely in all lessons. The teacher materials for every lesson contain exemplar responses. It is the responsibility of the teacher to enforce the habit of using terminology, symbols, and units appropriately.
7. Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.	Carnegie Learning's instructional materials focus on building a strong conceptual understanding of topics as opposed to memorization of facts. This understanding will lead to greater transfer and ability to solve non-routine problems. Throughout lessons students must be able to recognize numeric, geometric, and algebraic patterns. Students are also required to describe patterns verbally and symbolically. Students are reminded of their prior knowledge about the world and mathematical knowledge in developing new mathematical understanding. Take Note callout boxes in the text remind students of prior information.	**IN A2 10.4** The development of function is distributed over Algebra 1 and Algebra 2 as additional function families are developed. Over time, students generalize transformations to any function f(x). Students examine graphs of radical functions to generalize the effects of changing the equation on the graph.

8. Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	Through interactions with Carnegie Learning materials, students are expected to make observations, develop generalizations, and write formulas. Through classroom discourse, with in-depth accountable talk, and two-way interactions, whether among members of the whole class or small groups, teachers should stress the importance of attention to detail and the value of checking the reasonableness of solutions, both algebraically and in terms of a problem situation. The teacher's role is to facilitate classroom discussion and highlight important connections, efficient strategies, and conclusions.	**IN A2 7.3** Students calculate the value of *i*^*n* for *n* values between 1 and 10. They analyze the pattern and use it to derive strategy for calculating the value of *i*^*n* for any integer value of *n*.

Indiana Algebra II CCSS - Standards Coverage Response

STANDARD	Indiana Rating	Submitted Correlation	Response Detail	Carnegie Learning Rating Summary
N-CN.8 Extend polynomial identities to the complex numbers. For example, rewrite x^2+4 as $(x+2i)(x-2i)$.	0 – "Not covered"	Ch. 8	Determining a polynomial equation for a given set of roots is addressed in Indiana A2 8.6. Additional problems are provided in the skills practice. We do not have problems that require going from the polynomial equation to the factored form involving complex numbers. This standard is partially covered.	Moderate
A-APR.5 Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.	Not rated – "Can't find"		Binomial Probability Distribution is addressed in Indiana A1 15.2. The Binomial Theorem and Pascal's Triangle is addressed in Indiana Geometry 14.4 Problem 1.	Moderate* *Assumes use of other submitted course material
A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	0 – "Not sure if covered; can't find."		Linear programming involving linear function is addressed in Indiana A1 5.8. We do not have content in any existing materials that extends beyond linear functions.	Moderate* *Assumes use of other submitted course material

A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	Not rated – "Not found"	Ch. 6 Ch. 8	We do not explicitly use f(x) and g(x) notation when discussing a graphical approach to solving equations. More often, students transform the equation so that one side is equal to 0 and determine where the function crosses the x-axis. This is demonstrated in Indiana A2 6.2, 6.3, 8.2, and 9.6 in particular.	Moderate
F-BF.1b Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.	Not rated – "Not covered"	--	Students are exposed to numerous examples of functions that are created by composing or operating other functions, most notably in Indiana A2 Chapters 3 and 4. However, the functions are given as opposed to derived by the students. Students do create functions in Indiana A2 Chapter 5 through polynomial operations. In Indiana A2 6.7, students analyze scatter plots and create piecewise functions that consist of polynomial functions. In Indiana A2 9.6, students combine rational expressions to create equations that model work and mixture scenarios.	Strong

S-MD.6 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	No coverage	--	The interpretation of this standard is ambiguous at best. Throughout Indiana A2 Chapters 13-15, students analyze data collection methods and analysis to make "fair decisions" about valid or biased conclusions. While not included in the Indiana content submitted, fair games are addressed in national A1 11.9 and expected value is addressed in GM1 7.5.	Moderate
S-MD.7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	1.5 – "not covered"	--	As with S-MD.7, the interpretation of this standard is ambiguous at best. Throughout Indiana A2 Chapters 13-15, data and probability concepts are applied to real world situations including basketball, blood types, cell phone batteries, test scores, hockey goalies, and hybrid cars.	Strong
N-CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	2 – "Few problems"	Ch. 7	Addition, subtraction, multiplication, and division of complex numbers is addressed in Indiana A2 7.4. While the properties (commutative, associative, distributive) are not explicitly called out in the lesson, it is absolutely necessary that students understand the proper application of these properties and extension to the set of complex numbers. Review mentions "few problems". The purpose of the lesson is to introduce and develop concepts. The assignments and/or skills practice can be used to provide additional problems and necessary practice.	Strong

N-CN.7 Solve quadratic equations with real coefficients that have complex solutions.	1	Ch. 7	Quadratic equations with complex solutions is addressed throughout Indiana A2 chapter 8. Once complex numbers are introduced, (in Indiana A2 7.4), quadratic equations are treated as simple cases of polynomial equations. Solving a polynomial equation often requires solving a quadratic equation as part of the solution. For example, see Problem 1 Question 1e $x^4=1$. Solving this problem requires rewriting the equation as $\left(x^2+1\right)\left(x+1\right)\left(x-1\right)=0$. Calculating the solutions determined by the first factor requires solving a quadratic equation with complex solutions. These type of problems occur throughout the lessons, assignments, and skills practice for this chapter.	Strong
N-CN.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	1	Ch. 8	The Fundamental Theorem of Algebra is introduced in Indiana A2 8.3 and applied throughout Chapter 8. As with N-CN.7, once complex numbers are introduced, (in Indiana A2 7.4), quadratic equations are treated as simple cases of polynomial equations. In the lesson 2/9 problems involve quadratics and additional problems involving just quadratics can be found in the assignment and skills practice.	Strong
A-APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$ can be used to generate Pythagorean triples.	1	Ch. 8	Polynomial identities are not explicitly covered. However, students must be able to fluently operate with polynomial expressions throughout Indiana A2 Chapters 5, 8 and 9.	Moderate

F-BF.3 Identify the effect on the graph of replacing f(x) by f(x)+k, k f(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	~1.5 – "not developed"	Ch. 2 Ch. 6 Ch. 12	Transformations of the graphs of functions is addressed in: • Indiana A2 2.4 (logarithmic functions) • Indiana A2 6.1 & 6.3 (polynomial functions) • Indiana A2 10.4 (radical functions) • Indiana A2 12.2 (trig functions) Odd and even functions are introduced in Indiana A2 6.1.	Strong
A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V=IR to highlight resistance R.	1 - "only one problem exp/log functions"	Ch. 3	Solving literal equations is addressed in Indiana A1 3.7.	Strong* *Assumes use of other submitted course material
F-BF.4a Solve an equation of the form f(x)=c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x)=2x^3$ for x>0 or $f(x)=\frac{x+1}{x-1}$ for $x \neq 1$.	2	Ch. 2 Ch. 10	Inverses are addressed in Indiana A1 for linear and quadratic functions. Inverses are addressed in Indiana A2 2.1 (exponential and logarithmic functions) and 10.1 (radical functions).	Moderate
F-LE.4 For exponential models, express as a logarithm the solution to $ab^{ct}=d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate logarithms using technology.	1 - "only a few basic problems"	Ch. 3 Ch. 4	Logarithms and the properties of logarithms are used to solve both exponential and logarithmic equations throughout Indiana A2 Chapter 4.	Moderate